Fundamentals of Structural Design
Part of Steel Structures

Civil Engineering for Bachelors
133FSTD

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Syllabus of lectures

1. Introduction, history of steel structures, the applications and some representative structures, production of steel
2. Steel products, material properties and testing, steel grades
3. Manufacturing of steel structures, welding, mechanical fasteners
4. Safety of structures, limit state design, codes and specifications for the design
5. Tension, compression, buckling
6. Classification of cross sections, bending, shear, serviceability limit states
7. Buckling of webs, lateral-torsional stability, torsion, combination of internal forces
8. Fatigue
9. Design of bolted and welded connections
10. Steel-concrete composite structures
11. Fire and corrosion resistance, protection of steel structures, life cycle assessment

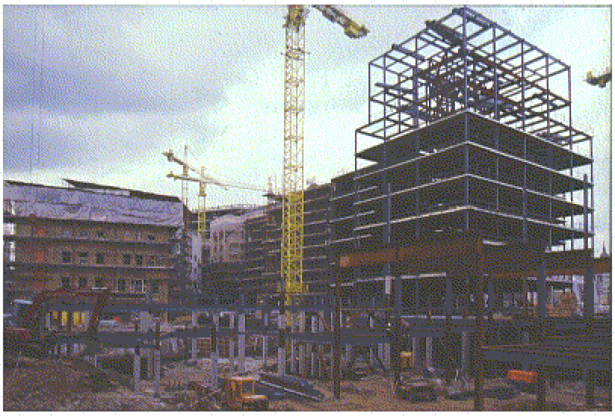
Scope of the lecture

- Tension and compression elements - examples
- Design of elements loaded in tension
- Design of elements loaded in compression
  - Behaviour of perfect element
  - Real element
  - Built-up element

Elements loaded by axial force

- Tension, compression or alternating load
- Frequently designed for:
  - trusses
  - ties (tension)
  - columns (compression)
  - bracing diagonals (tension and compression)

Elements loaded by axial force



Multi-storey building

The columns of a multi-storey buildings are typical example of structural elements loaded in compression. No bending is introduced as the connections are usually designed as simple connection.

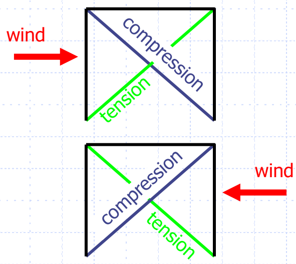
Elements loaded by axial force

Industrial building - technological platform

diagonals of the bracing

The columns are loaded in compression. No bending is introduced as the connections are usually as simple connection.

The resistance to horizontal load (e.g. wind load) is ensured by diagonal bracing, the diagonals are loaded in tension and compression, but this may alternate depending on the wind direction.



Elements loaded by axial force

Trusses of a single-storey industrial building

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Elements loaded by axial force

Trusses of a single-storey industrial building

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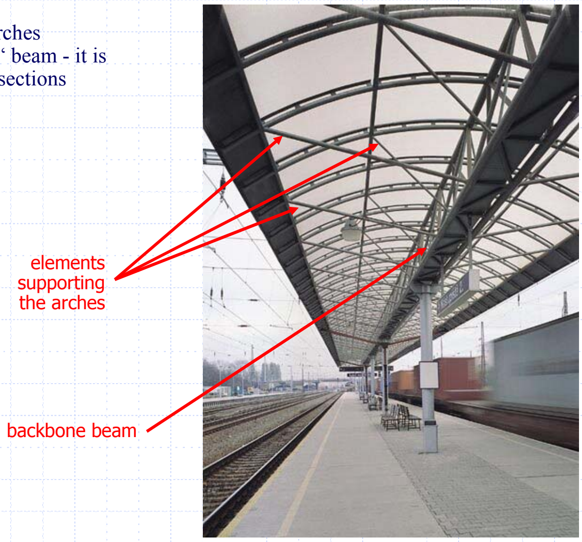
Elements loaded by axial force

Roof bracing of a single-storey industrial building

Elements loaded by axial force

The roof is made from arches supported on “backbone“ beam - it is truss made from hollow sections



Railway platform

Elements loaded by axial force

Trusses of Sazka Arena (now O2 Arena) in Praha Vysočany

Circular plan, diameter 135 m

The roof is made from trusses and pre-stressed ties all connected to the central ring

Elements loaded by axial force

Various types of towers

Elements loaded by axial force

Žďákovský Bridge (Vltava river) - South Bohemia

Bracing and columns supporting the bridge deck of the Žďákovský Bridge spanning 330 m (Vltava river, South Bohemia)

Elements loaded by axial force

The bridge deck is suspended on cables to the arch.

The cables are special elements loaded in tension as their behaviour is different from "standard" elements: the cables require pre-stressing and their response is non-linear, requiring non-linear analysis of the structure.

Elements loaded by axial force

Scope of the lecture

- Tension and compression elements - examples
- ➡ Design of elements loaded in tension
- Design of elements loaded in compression
  - Behaviour of perfect element
  - Real element
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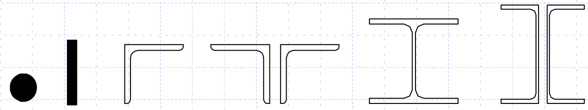
Tension elements

Typical cross-sections



Connection is important - the choice of cross section might be influenced by the way it I connected to the other elements

Resistance of tension elements

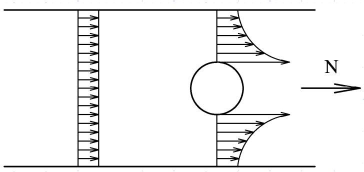
Resistance:

- full cross-section (elastic resistance)

$$N_{pl,Rd} = \frac{A\, f_y}{\gamma_{M0}}$$

- net cross-section at holes for fasteners (ultimate resistance)

$$N_{u,Rd} = \frac{0{,}9\, A_{net}\, f_u}{\gamma_{M2}}$$



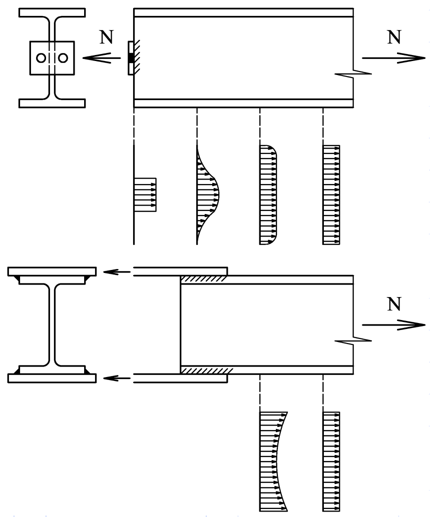
Stress distribution in element loaded in tension

Elements loaded in tension

Care should be taken about the stress distribution near the connection

Uniform stress distribution can be found “far” from the connection

Non-uniform stress distribution is found near the connection when some parts of the element are not connected



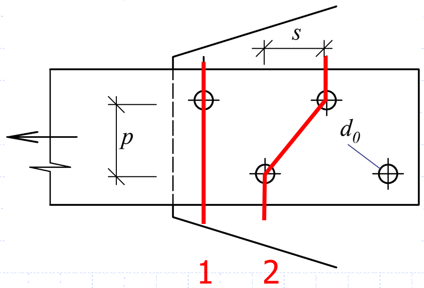
Net area A_{net}

- Failure along the straight line perpendicular to axis of the element (line 1)

$$A_{net} = A - n\,t\,d_0$$

- Failure along the zig-zag line for staggered holes (line 2)

$$A_{net} = A - t\left(n\,d_0 - \sum \frac{s^2}{4\,p}\right)$$



Scope of the lecture

Tension and compression elements - examples

Design of elements loaded in tension

Design of elements loaded in compression

- Behaviour of perfect element
- Real element
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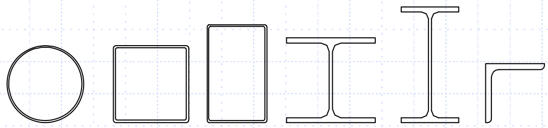
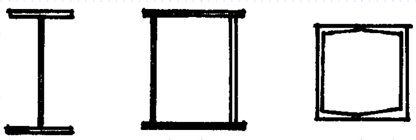
Compression elements

Cross-sections:

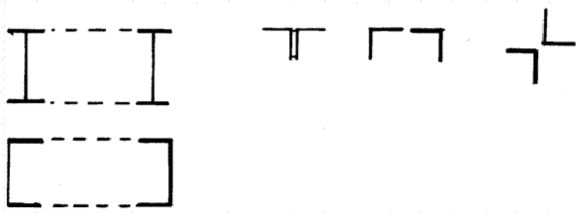
- solid
  - hot-rolled



  - welded



- built-up



Behaviour of compression elements

- Short elements (quite rare)
  compression stress of cross-section is checked
  yield limit should not be exceeded
- Long elements (all ordinary elements)
  buckling resistance needs to be evaluated

Buckling

- Stability phenomena
  - buckling occurs before f_y is reached in the cross-section
  - the most frequent reason for collapse of steel structures
- Stability problems need to be considered for two types of elements:
  - Perfect (ideal) element
    - no imperfections
    - only theoretical, does not appear in reality
    - theoretical solution leads to stability problem
  - Real element
    - different types of imperfection exist
    - real elements in everyday life
    - leads to buckling resistance

Scope of the lecture

Tension and compression elements - examples

Design of elements loaded in tension

Design of elements loaded in compression

- Behaviour of perfect element
- Real element
- Built-up element

Stability of perfect element

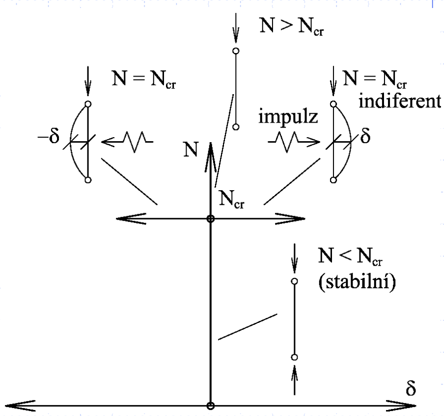
Perfect element is:

- Straight element (no bow shape)
- Pinned ends (perfect hinges)
- Centric loading
- No imperfections (residual stresses, etc.)

Solution was derived by Euler in 1744

The element is stable for all loads smaller than the critical load

Indiferent equilibrium is achieved when the critical load is reached, i.e. very small lateral load leads to loss of stability



Critical (Euler's) load

The critical load is obtained from differential equation

$$y'' + \frac{N}{E\,I}\,y = 0$$

and the boundary conditions

$$x = 0 \quad \rightarrow \quad y = 0$$

$$x = L \quad \rightarrow \quad y = 0$$

Solution

$$y = C_1\, sin\,(a\,x) + C_2\, cos\,(a\,x)$$

Applying the boundary conditions:

$$C_2 = 0$$

$$\sin\,(a\,L) = 0 \quad \rightarrow \quad a\,L = \pi, 2\pi, 3\pi, \ldots$$

Critical force

$$N_{cr} = \frac{\pi^2 E\,I}{L^2}$$

Critical (Euler's) load

Critical force

$$N_{cr} = \frac{\pi^2 E\,I}{L^2}$$

Critical stress

$$\sigma_{cr} = \frac{N_{cr}}{A} = \frac{\pi^2 E\,I}{A\,L^2}$$

New parameter - slenderness of the element is introduced

$$\lambda = \pi\sqrt{\frac{E}{\sigma_{cr}}} = \pi\sqrt{\frac{E}{\frac{A\,L^2}{\pi^2\,E\,I}}} = \sqrt{\frac{L^2}{\left(\frac{I}{A}\right)}} = \frac{L}{i}$$

and new section parameter - radius of gyration is used in evaluation of the slenderness

$$i = \sqrt{\frac{I}{A}}$$

Buckling length

Critical force

$$N_{cr} = \frac{\pi^2 E\, I}{{L_{cr}}^2}$$

Slenderness

$$\lambda = \frac{L_{cr}}{i}$$

Buckling length is introduced to take into account other boundary conditions (it relates the critical load of the element to critical load of element with hinges at both ends)

$$L_{cr} = \beta\, L$$

It can be derived from Euler’s formula and corresponding boundary conditions

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Buckling length

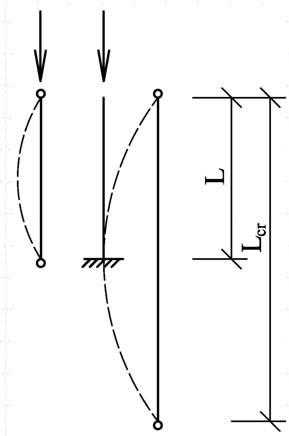
Euler’s formula

$$y'' + \frac{N}{E\, I}\, y = 0$$

and boundary conditions for cantilever

$$x = 0 \quad \rightarrow \quad y = 0$$

$$x = 0 \quad \rightarrow \quad y' = 0$$



Critical load of cantilever

$$N_{cr} = \frac{\pi^2 E\, I}{4\, L^2}$$

Critical length

$$L_{cr} = 2\, L \quad \rightarrow \quad \beta = 2$$

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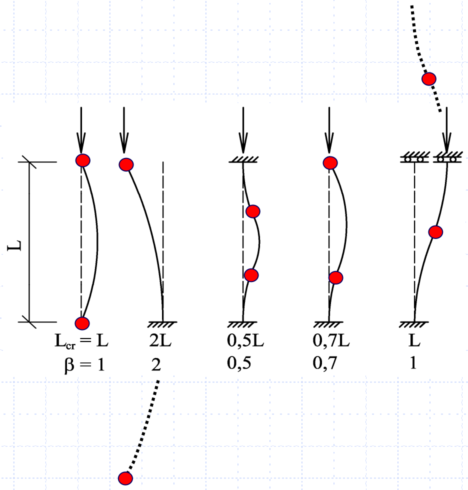
Basic boundary conditions for buckling

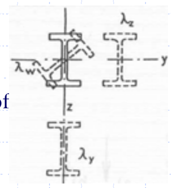
buckling (critical) length = distance between 2 points of contraflexure



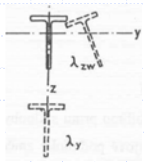
Buckling of element

Buckling can take these modes:

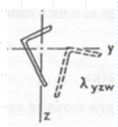
- Double axis symmetric sections
  - Flexural buckling – deformation perpendicular to principal axes of the section
  - torsional buckling– no lateral deformation but the element is twisted
  - slenderness λ_y, λ_z, λ_{zw}



- Uni-axial symmetrical sections
  - flexural buckling – lateral deformation in the plane of symmetry
  - flexural-torsional buckling – lateral deformation perpendicular to the plane of symmetry and torsion
  - slenderness λ_y, λ_{yzw}



- Non-symmetrical sections
  - flexural-torsional buckling – lateral deformation in general direction and torsion
  - Slenderness λ_{yzw}
  - It is taken into account in simplified form



Buckling length

Buckling lengths must be considered in two different planes (usually called "in plane" and "out of plane")

Generally: $L_{cr,y} \neq L_{cr,z}$

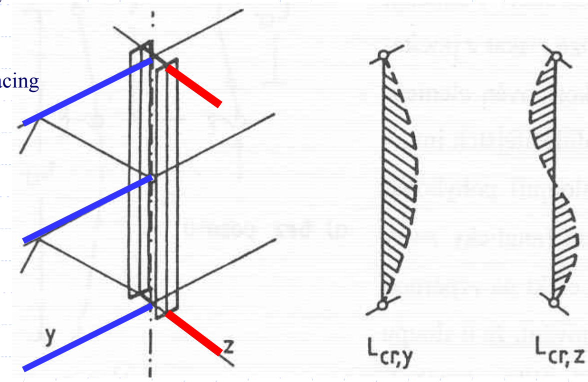
Example: column of the bracing

In plane of the bracing

$L_{cr,z} = L / 2$

Out of plane of the bracing

$L_{cr,y} = L$



Other cases of buckling

Column with cantilever end

Two-bay column

The precise evaluation of buckling length is more complicated

Buckling length of trusses

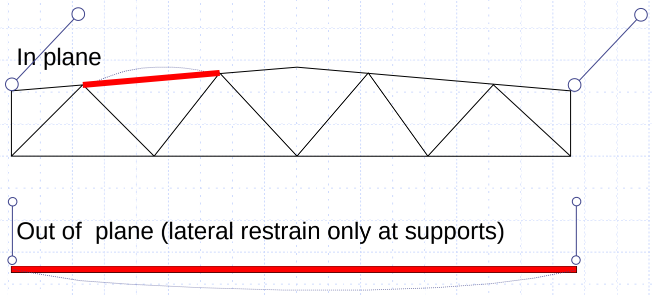
Chords

- In plane of the truss

  Buckling length = distance between the joints

- Out of plane

  Buckling length = distance between points of lateral restraint



Buckling length of trusses

Diagonals

- In plane of the truss

  In-plane stiffness of the plate reduces the buckling length

  Buckling length = distance of centers of the connections of the element to the plates

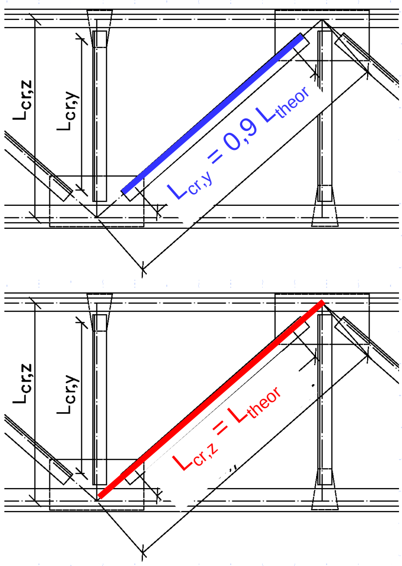
  Approximately $L_{cr,y} = 0,9\ L_{theor}$.

- Out of plane

  Thin plate can be bended, does not reduce the buckling length

  Buckling length = theoretical length of the elements

  $L_{cr,z} = L_{theor}$



Buckling length of frames

Depends on:

- boundary conditions
- loading
- stiffness ratio of beams and columns

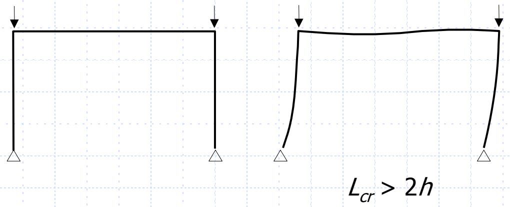
Frames:

- sway frames
  Horizontal movement of the beam is not restrained
- non-sway frames
  Horizontal movement of the beam is not restrained

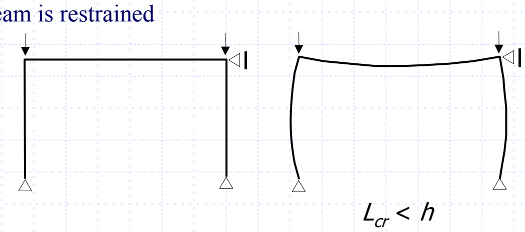
Buckling lengths of frames

Pinned frame

Sway frame
Horizontal movement of the beam is not restrained



Non-sway frame
Horizontal movement of the beam is restrained



Scope of the lecture

- Tension and compression elements - examples
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Buckling resistance of real element

Real elements have imperfections

- Geometrical imperfections
  - initial curvature (bow shape) of the element axis,
  - excentricity of the loading,
  - deviation from the theoretical shape of the cross-section
- Material imperfections
  - Residual stresses due to the welding, straightening or cooling
- Structural imperfections
  - Imperfect function of hinges or fixed connections

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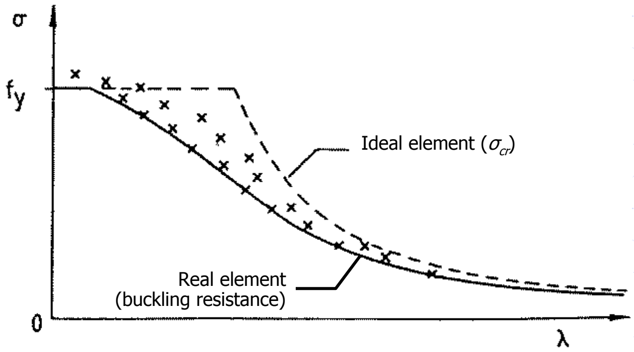
Results of experiments of compression members



Influence of geometrical imperfections

It is assumed the initial imperfection take the following shape

$$y_0 = e_0 \, sin \frac{\pi \, x}{L}$$

Differential equation of the deformed element

$$\frac{d^2 y}{d\, x^2} + \frac{N \left(y + y_0\right)}{E\, I} = 0$$

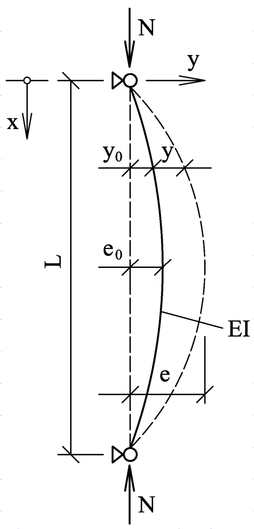
Boundary conditions

$$x = 0 \quad \rightarrow \quad y + y_0 = 0$$

$$x = 0 \quad \rightarrow \quad y + y_0 = 0$$

Solution

$$y = \frac{e_0}{\frac{N_{cr}}{N} - 1} \, sin \frac{\pi \, x}{L}$$

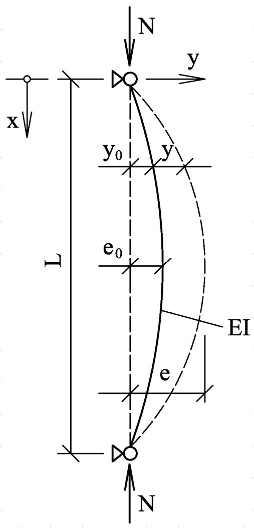


Influence of geometrical imperfections

the deformation for $x = \dfrac{L}{2}$ is equal to

$$e = e_0 + \frac{e_0}{\frac{N_{cr}}{N} - 1} = \frac{e_0}{1 - \frac{N}{N_{cr}}}$$

where the multiplication factor $\left(1 - \dfrac{N}{N_{cr}}\right)$ indicates the deformation increase with increasing load N and approaches to infinity when n is approaching to N_{cr}



Influence of geometrical imperfections

Strength of the element is reached when the stress at mid-length of the column reach the yield limit f_y

$$\sigma = \frac{N}{A} + \frac{M}{W} = \frac{N}{A} + \frac{N\,e}{W} = f_y$$

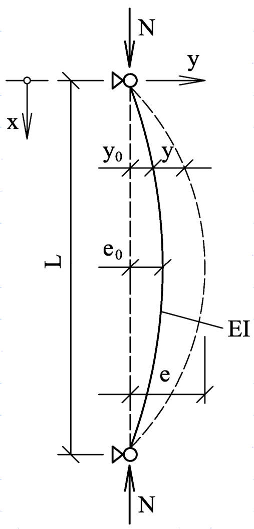
Stress representing the buckling strength $\sigma_b = \dfrac{N}{A}$ is substituted

$$\sigma_b + \sigma_b \frac{e\,A}{W} = f_y$$

The same equation at mid-length of the element, where the

deformation $e = \dfrac{e_0}{1 - \dfrac{N}{N_{cr}}}$ is equal to

$$\sigma_b + \sigma_b \frac{e_0}{1 - \frac{\sigma_b}{\sigma_{cr}}} \frac{A}{W} = f_y$$



Influence of geometrical imperfections

The equation

$$\sigma_b + \sigma_b \frac{e_0}{1-\dfrac{\sigma_b}{\sigma_{cr}}} \frac{A}{W} = f_y$$

After some algebra Ayrton - Perry formula is obtained

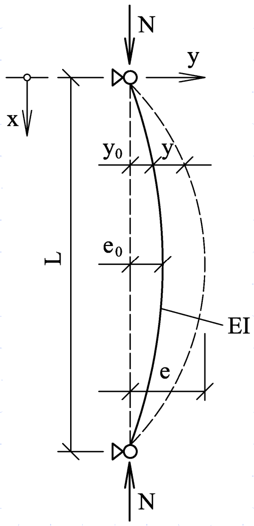
$$(\sigma_{cr}-\sigma_b)(f_y-\sigma_b)=\sigma_b\ \sigma_{cr}\ e_0 \frac{A}{W}$$

which can be re-arranged into following

$$\left(\frac{\sigma_{cr}}{f_y}-\chi\right)(1-\chi)=\frac{\eta\ \chi\ \sigma_{cr}}{f_y}$$

where χ is buckling reduction factor $\chi = \dfrac{\sigma_b}{f_y}$

and $\eta = e_0 \dfrac{A}{W}$



Derivation of buckling reduction factor

The Ayrton-Perry formula

$$\left(\frac{\sigma_{cr}}{f_y}-\chi\right)(1-\chi)=\frac{\eta\ \chi\ \sigma_{cr}}{f_y}$$

can be further simplified by substituting $\bar{\lambda}^2 = \dfrac{f_y}{\sigma_{cr}}$ where $\bar{\lambda} = \dfrac{\lambda}{\lambda_1}$ and $\lambda_1 = \pi \sqrt{\dfrac{E}{f_y}}$

$$(1-\chi\ \bar{\lambda}^2)(1-\chi)=\eta\ \chi$$

$$\bar{\lambda}^2\ \chi^2 - \chi\left(\bar{\lambda}^2+\eta+1\right)+1=0$$

The formula above is used to derive the buckling reduction factor χ, in fact it is quadratic equation

Buckling reduction factor

All the imperfections are expressed as geometrical imperfections – e_0

α is imperfection factor, it includes "the amount" of imperfections (it was obtained from tests and numerical modeling)

$$\phi = 0{,}5\left[1 + \alpha\left(\bar{\lambda} - 0{,}2\right) + \bar{\lambda}^2\right]$$

$$\chi = \frac{1}{\phi + \sqrt{\phi^2 - \bar{\lambda}^2}} \leq 1$$

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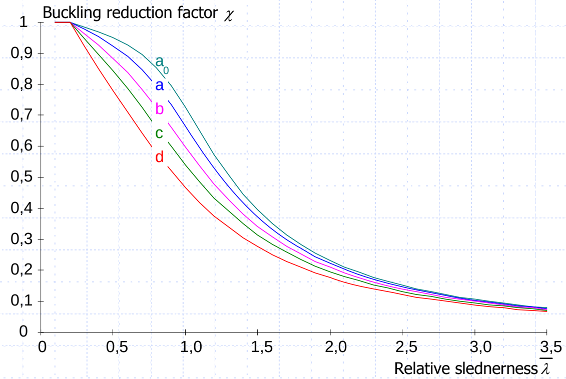
Buckling curves

Imperfection factor α range from 0,15 to 0,76 resulting in 5 buckling curves (the curve a_0 is used only for some elements made from steel S460)

These are used for corresponding section shapes

Include the amount of imperfections introduced during manufacturing



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Buckling resistance of compressed element

Design buckling resistance

$$N_{b,Rd} = \frac{\chi\, A\, f_y}{\gamma_{M1}}$$

buckling reduction factor χ should be evaluated for the corresponding slenderness λ

$$\lambda = \frac{L_{cr}}{i}$$

relative slenderness

$$\overline{\lambda} = \frac{\lambda}{\lambda_1} \quad \text{where} \quad \lambda_1 = \pi \sqrt{\frac{E}{f_y}}$$

χ is evaluated using the imperfection factor α (depends on cross-section type)

$$\chi = \frac{1}{\phi + \sqrt{\phi^2 - \overline{\lambda}^2}} \le 1 \quad \text{where} \quad \phi = 0{,}5\left[1 + \alpha\left(\overline{\lambda} - 0{,}2\right) + \overline{\lambda}^2\right]$$

Scope of the lecture

Tension and compression elements - examples

Design of elements loaded in tension

Design of elements loaded in compression

- Behaviour of perfect elements
- Real elements
- ➡ Built-up elements

Built up elements

The built-up elements are usually used for:

- columns
- internal elements of trusses

Reasons:

- easy connection - gap
- structural analysis – increased stiffness of the element

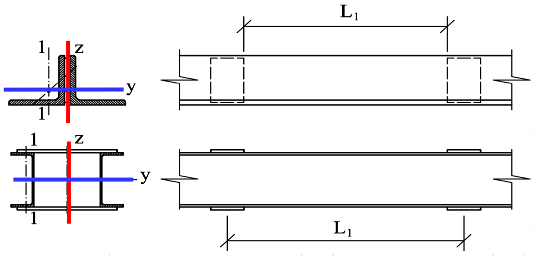
Battened column

Battened column composed from two channels

Built up elements

Buckling in two directions must be considered

- perpendicular to mass axis (y-axis at the picture)
  - the resistance check is carried out as for "standard" elements
- perpendicular to non-mass axis (z-axis at the picture)
  - influence of shear deflection of the connecting element (battens) and buckling of partial element between battens needs to be considered
  - completely different procedure is adopted
  - it is not included in course of STS1



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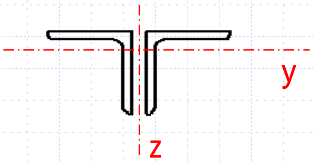
Battened elements from angles

Special case:



- element is made from equal leg angles
- the buckling lengths $L_{cr,y}$ and $L_{cr,z}$ are (approximately) equal
- at least two battens are placed at thirds of element length (and another two are at the ends)

⇒ buckling perpendicular to the mass axis (y-axis) governs, no need to calculate buckling resistance for buckling perpendicular to non-mass axis (z-axis)

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Thank you for your attention